DETERMINATION OF CURRENT FLOW IN A PLASMA BY A SIMULATION METHOD

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A method is proposed which, for specific assumptions, allows us to determine the density distribution of a constant current flowing between electrodes in a plasma for plane parallel or radially symmetric electric and magnetic fields, allowing for anisotropic conductivity.

NOTATION

e_r, e_θ, e_z are the unit vectors in a cylindrical coordinate system; E, E_r, E_z are the electric field strength vector and its components; V is the electric field potential; H, H_r, H_θ, H_z are the magnetic field strength and its components; j is the current density vector; e is the electron charge; m is the electron mass; c is the velocity of light; τ is the momentum transfer time; σ_0 is the normal plasma conductivity; ω_e is the electron cyclotron frequency; h is the unit vector in the direction of the magnetic field.

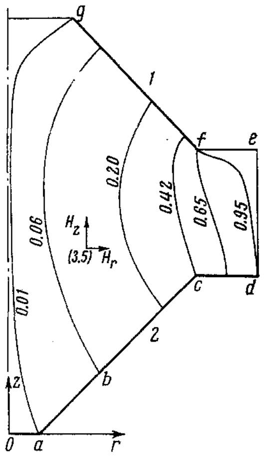

Fig. 1

We shall assume that a) the electrodes are made of perfectly conducting material and the walls of the enclosure of insulating material, b) the self-field of the current flowing in the plasma may be neglected.

The equation for Ohm's law for a constant current in a stationary uniform plasma has the form [1]

$$\mathbf{j} + \omega_e \tau\, [\mathbf{j}\mathbf{h}] = \sigma_0 \mathbf{E} \qquad (\omega_e = eH/mc). \tag{1}$$

Thus the conductivity tensor of a plasma situated in a magnetic field of field strength $\mathbf{H} = \{H_r, H_\theta, H_z\}$ has the form

$$|\sigma| = \sigma_0 \|\Omega_{\lambda,\mu}\| \qquad (\lambda, \mu = 1, 2, 3), \tag{2}$$

$$\Omega_{11} = \Omega_0 (1 + \omega_{er}{}^2 \tau^2), \qquad \Omega_{12} = \Omega_0 (\omega_{er}\omega_{e\theta}\tau^2 - \omega_{ez}\tau),$$

$$\Omega_{13} = \Omega_0 (\omega_{er}\omega_{ez}\tau^2 + \omega_{e\theta}\tau), \qquad \Omega_{21} = \Omega_0 (\omega_{er}\omega_{e\theta}\tau^2 + \omega_{ez}\tau),$$

$$\Omega_{22} = \Omega_0 (1 + \omega_{e\theta}{}^2\tau^2), \qquad \Omega_{23} = \Omega_0 (\omega_{e\theta}\omega_{ez}\tau^2 - \omega_{er}\tau),$$

$$\Omega_{31} = \Omega_0 (\omega_{er}\omega_{ez}\tau^2 - \omega_{e\theta}\tau), \qquad \Omega_{32} = \Omega_0 (\omega_{\theta e}\omega_{ez}\tau^2 + \omega_{er}\tau),$$

$$\Omega_{33} = \Omega_0 (1 + \omega_{ez}^2\tau^2), \qquad \Omega_0 = (1 + \omega_e^2\tau^2)^{-1},$$

$$\omega_{er} = \frac{eH_r}{mc}, \qquad \omega_{ez} = \frac{eH_z}{mc}, \qquad \omega_{e\theta} = \frac{eH_\theta}{mc}. \tag{3}$$

For an electric field of field strength

$$\mathbf{E} = \{E_r, 0, E_z\} = -\operatorname{grad} V = -\mathbf{e}_r \frac{\partial V}{\partial r} - \mathbf{e}_z \frac{\partial V}{\partial z} \tag{4}$$

the current density in the plasma is

$$\mathbf{j} = \sigma_0 \{\Omega_{11}E_r + \Omega_{13}E_z\}\, \mathbf{e}_r + \sigma_0 \{\Omega_{21}E_r + \Omega_{23}E_z\}\, \mathbf{e}_\theta + $$
$$+ \sigma_0 \{\Omega_{31}E_r + \Omega_{33}E_z\}\, \mathbf{e}_z. \tag{5}$$

In the steady state $\nabla \mathbf{j} = 0$. Since in an axially symmetric field $\partial/\partial\theta = 0$ it follows from (4) and (5) that

$$\Omega_{11} \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \Omega_{33}\frac{\partial^2 V}{\partial z^2} + \Omega_{13}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial z}\right) +$$
$$+ \Omega_{31}\frac{\partial^2 V}{\partial r\,\partial z} + \left(\frac{\partial\Omega_{31}}{\partial z} + \frac{\partial\Omega_{11}}{\partial r}\right)\frac{\partial V}{\partial r} + \left(\frac{\partial\Omega_{13}}{\partial r} + \frac{\partial\Omega_{33}}{\partial z}\right)\frac{\partial V}{\partial z} = 0. \tag{6}$$

In the general case of a nonuniform field this is an equation with variable coefficients. In a sufficiently small volume of space where the field may be taken to be uniform $\partial H/\partial r = \partial H/\partial z = 0$ and Eq. (6) assumes the form

$$\Omega_{11k}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \Omega_{33k}\frac{\partial^2 V}{\partial z^2} +$$
$$+ \Omega_{13k}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial z}\right) + \Omega_{31k}\frac{\partial^2 V}{\partial r\,\partial z} = 0. \tag{7}$$

Here Ω_{11k}, Ω_{33k}, Ω_{13k}, Ω_{31k} are constants obtained for the mean value of H in the k-th element of space. Making the following change of variables

$$\eta = \frac{\sqrt{4\Omega_{11k}\Omega_{33k} - (\Omega_{31k} + \Omega_{13k})^2}}{2\Omega_{11k}}\, r,$$
$$\xi = z - \frac{\Omega_{31k} + \Omega_{13k}}{2\Omega_{11k}}\, r \tag{8}$$

enables us to reduce Eq. (7) to the canonical form [2]

$$\frac{4\Omega_{11k}\Omega_{33k} - (\Omega_{31k} + \Omega_{13k})^2}{4\Omega_{11k}}\left\{\frac{\partial^2 V}{\partial \xi^2} + \frac{\partial^2 V}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial V}{\partial \eta}\right\} +$$
$$+ \frac{(\Omega_{31k} - \Omega_{13k})\sqrt{4\Omega_{11k}\Omega_{33k} - (\Omega_{31k} + \Omega_{13k})^2}}{4\Omega_{11k}}\frac{\partial V}{\partial \xi} = 0. \tag{9}$$

Thus if the magnetic field component H_θ can be neglected (i.e., the self-field of the current in the plasma may be neglected), then

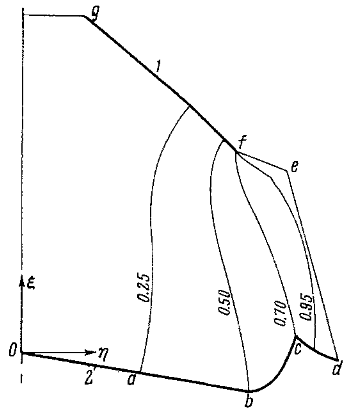

Fig. 2

$\Omega_{31} = \Omega_{13}$ and Eq. (9) becomes Laplace's equation,

$$\Delta V = 0, \tag{10}$$

the solution of which may be obtained on an analog machine and gives the current distribution in the plasma [3].

The boundary conditions in the (η, ξ) plane remain the same as in the (r, z) plane while the boundary itself changes. The point $A(m, l)$ in the rz plane passes to the point $A'(m', l')$ in the (η, ξ) plane. Here

$$m' = \int_0^m \frac{\sqrt{\Omega_{11}\Omega_{33} - \Omega_{13}^2}}{\Omega_{11}} dr = \frac{m}{n} \sum_{k=1}^{n} \frac{\sqrt{\Omega_{11k}\Omega_{33k} - \Omega_{13k}^2}}{\Omega_{11k}} \Delta r,$$

$$l' = l - \int_0^m \frac{\Omega_{13}}{\Omega_{11}} dr = l - \frac{m}{n} \sum_{k=1}^{n} \frac{\Omega_{13k}}{\Omega_{11k}} \Delta r. \qquad (11)$$

where n is the number of separations of the abscissa of point $A(m, l$ for equal intervals Δr.

In the case of a uniform field directed along the Z axis, $\mathbf{H} = \{0, 0, \mathbf{H}_z\}$, transformation of the region leads to compression along the Z axis with coefficient k_z [4]:

$$1 / k_z = \sqrt{1 + \omega_e^2 \tau^2}. \qquad (12)$$

We now give results of the solution of a problem of this type. Two electrodes—a cathode 1 and anode 2 are placed in the cylindrical chamber of Fig. 1. The chamber is situated in a radially symmetric magnetic field increasing in the direction of the anode (the solenoid is not shown in Fig. 1). The magnetic field strength is measured at intervals of 1 cm by a Minsk-1 machine. The pressure in the chamber was p = 0.1 mm Hg, and T_e = 25 000° K.

Figure 1 shows the distribution of streamlines in the absence of a magnetic field. Figure 2 gives a transformation of both chamber and current distribution in the (η, ξ) plane for a field strength of H = 40 Oe at the point r = 0, z = 0 and $\mathbf{H}_r$ = 4 Oe, H_z = 18 Oe at the point r = 3, z = 5. Figure 3 gives a transformation of the chamber, and current distribution for H = 200 Oe, H_r = 20 Oe, H_z = 90 Oe at the same points.

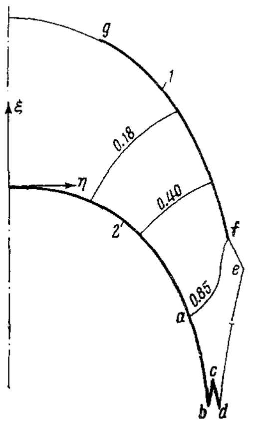

Fig. 3

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